

Second Term Test - 2020

Combined Mathematics I - Part A - Grade 12

1) $(1-k)x^2 + x + k = 0$

Consider,

$$\begin{aligned}\Delta_x &= b^2 - 4ac, \\ &= 1 - 4(1-k)k \quad (5) \\ &= 4k^2 - 4k + 1 \\ &= 4\left\{\left(k - \frac{1}{2}\right)^2 + \frac{15}{4}\right\} > 0 \quad (5)\end{aligned}$$

Roots of the eqⁿ α and β ,

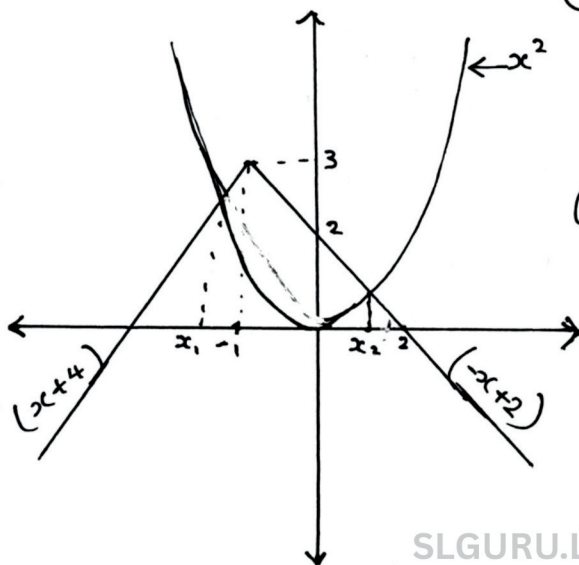
$$\alpha + \beta = \frac{-1}{1-k}$$

$$(5) = \frac{1}{k-1} < 0 \quad (\because 0 < k < 1)$$

$$\alpha\beta = \frac{k}{1-k} > 0 \quad (5)$$

Therefore, eqⁿ has negative real roots. (5)

2) $3 - |x+1| < x^2$



$$\begin{aligned}(5) \quad \underline{x_1} \quad (5) \quad \underline{x_2} \\ x^2 &= x+4 & x^2 &= 2-x \\ x^2 - x - 4 &= 0 & x^2 + x - 2 &= 0 \\ x &= \frac{1 \pm \sqrt{17}}{2} & (x+2)(x-1) &= 0 \\ (5) \quad \therefore x_1 &= \frac{1 - \sqrt{17}}{2} & x_2 &= 1; x_3 = -2 \\ & & (5) &\end{aligned}$$

solution.

$$x < \frac{1 - \sqrt{17}}{2} \quad \text{or} \quad x > 1$$

$$03) 3^{2x+1} - 3^{x+4} + 3^3 = 3^x$$

$$3(3^x)^2 - 3^4(3^x) - 3^x + 3^3 = 0$$

$$3(3^x)^2 - 82(3^x) + 27 = 0 \quad (5)$$

$$\text{Let } 3^x = t,$$

$$(5) 3t^2 - 82t + 27 = 0$$

$$(t^2 - 2)(3t - 1) = 0 \quad (5)$$

$$t = 27 \quad \text{or} \quad t = \frac{1}{3}$$

$$3^x = 27, \quad 3^x = 3^{-1}$$

$$\underline{x = 3} \quad (5) \quad \underline{x = -1} \quad (5)$$

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$$04) \lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{\sin[\pi(1 - \sin^2 x)]}{x^2} \quad (5)$$

$$\lim_{x \rightarrow 0} \frac{\sin \pi \sin^2 x}{x^2} \quad (5)$$

$$\lim_{x \rightarrow 0} \frac{\sin \pi \sin^2 x}{\pi \sin^2 x} \times \frac{\pi \sin^2 x}{x^2} \quad (5)$$

$$= \lim_{x \rightarrow 0} \frac{\sin \pi \sin^2 x}{\pi \sin^2 x} \times \pi \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2}$$

$$= 1 \times \pi \times (1)^2 \quad (5)$$

$$= \underline{\underline{\pi}} \quad (5)$$

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$$05) \log_3 x + \log_3 y = 3 \text{ --- (1)}$$

$$\log_y x = 2 \text{ --- (2)}$$

From (1),

$$\log_3 xy = 3 \text{ (5)}$$

$$xy = 27 \text{ --- (3)}$$

From (2);

$$x = y^2 \text{ --- (4)}$$

(5)

(3) and (4), (5)

$$y^3 = 27$$

$$\underline{y = 3} \text{ (5)} \quad \underline{x = 9} \text{ (5)}$$

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$$06) \text{ Let; } f(x) = x^3 + ax^2 + b$$

$$g(x) = ax^3 + bx^2 + x - a$$

Let, $(x - \alpha)$, common factor of $f(x)$ and $g(x)$

$$\alpha^3 + a\alpha^2 + b = 0 \text{ --- (1) (5)}$$

$$a\alpha^3 + b\alpha^2 + \alpha - a = 0 \text{ --- (2) (5)}$$

$$a \times (1) - (2) \Rightarrow$$

$$(5) (\alpha^2 - b)\alpha^2 - \alpha + ab + a = 0$$

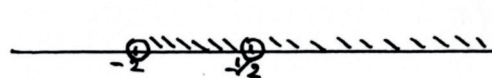
$$(b - \alpha^2)\alpha^2 + \alpha - a(1 + b) = 0 \text{ (5)}$$

$\Rightarrow (b - \alpha^2)x^2 + x - a(1 + b)$ has common linear factor $(x - \alpha)$. (5)

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07). $f(x) = \frac{1}{\sqrt{x+2}}$; $x \geq -2$ $g(x) = 2x+1$,

i) $\frac{f}{g} = \frac{1}{\sqrt{x+2}} (2x+1)$ (5)

 (5)

Domain of $\frac{f}{g}$; $(-2, 1/2) \cup (1/2, \infty)$ (5)

ii). $\left(\frac{f}{g}\right)(0) = \frac{1}{\sqrt{0+2}} (2 \times 0 + 1)$ (5) $= \frac{1}{\sqrt{2}}$ (5)

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08). $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ $\begin{matrix} \alpha \\ \beta \end{matrix}$
roots of the eqⁿ,

$x = \frac{6a \pm \sqrt{36a^2 - 4(2 - 2a + 9a^2)}}{2}$ (5)

$x = \frac{6a \pm \sqrt{8a-8}}{2}$
 $= 3a \pm \sqrt{2a-2}$ (5)

But, $\alpha, \beta > 3$ $\alpha + \beta > 6$,
 $a > 1$

$\therefore 3a - \sqrt{2a-2} > 3$ (5)

$3(a-1) > \sqrt{2a-2}$

$(9a-11)(a+1) > 0$ (5)

$9(a-1)^2 > 2a-2$ $(9a-11) > 0$

$9a^2 - 20a + 11 > 0$ $a > \frac{11}{9}$ (5)

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9) $\sec \alpha + \tan \alpha = p$

(5) $\tan \alpha = p - \sec \alpha$

$\Rightarrow \tan^2 \alpha = (p - \sec \alpha)^2$ (5)

$(\tan \alpha - p)^2 = 1 + \tan^2 \alpha$ (5)

~~$\tan^2 \alpha - 2p \tan \alpha + p^2 = 1 + \tan^2 \alpha$~~

$\tan \alpha = \frac{p^2 - 1}{2p}$ (5)

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10) $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$

Let,

$\alpha = \sin^{-1}\left(\frac{5}{x}\right)$, $\beta = \sin^{-1}\left(\frac{12}{x}\right)$, γ

$\sin \alpha = \frac{5}{x}$

$\sin \beta = \frac{12}{x}$

Then,

$\alpha + \beta = \frac{\pi}{2}$

$\alpha = \frac{\pi}{2} - \beta$ (5)

$\sin \alpha = \sin\left(\frac{\pi}{2} - \beta\right)$

$\sin \alpha = \cos \beta$ (5)

$\frac{5}{x} = \frac{\sqrt{x^2 - 144}}{x}$

; $x \neq 0$,

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$x^2 - 144 = 25$

$x = \pm 13$ ($\alpha, \beta < \frac{\pi}{2}$)

(5)

solution $x = 13$ (5)

$$f(1) = 1 + p + r = -9$$

$$f(0) = \underline{r = -8} \quad (5)$$

$$\underline{p = -2} \quad (5')$$

$$\begin{aligned} x^4 - 2x^2 - 8 &\equiv (ax^2 + b)^2 + c \\ &= a^2x^4 + 2abx^2 + b^2 + c \end{aligned}$$

Equating coefficients,

$$x^4 \rightarrow 1 = a^2 \quad \text{--- (1)}$$

$$\begin{aligned} x^2 &\rightarrow 0 = 2ab \\ ab &= -1 \quad \text{--- (2)} \end{aligned} \quad (10)$$

$$-8 = b^2 + c \quad \text{--- (3)}$$

(5) a = 1 b = -1 (5) c = -9 (5)

$$\begin{aligned} \therefore f(x) &= (x^2-1)^2 - 9 \\ &= (x^2-1)^2 - 3^2 \quad (5) \\ &= (x^2-1-3)(x^2-1+3) \\ &= (x^2-4)(x^2+2) \quad (5) \\ &= (x-2)(x+2)(x^2+2) \end{aligned}$$

Hence; real roots of the eqⁿ $\frac{x=2}{(5)}$ $\frac{x=-2}{(5)}$

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b). Let $h(x) = (p-1)x^2 - 4x + p-1$

$\forall x \in \mathbb{R}, h(x) > 0;$

$(p-1) > 0$ (5) and $\Delta_x < 0$ (5)

$\Delta_x = 16 - 4(p-1)(p-1) < 0$

$4 - (p-1)^2 < 0$ (5)

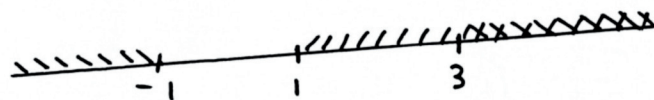
$(2-p+1)(2+p-1) < 0$

(5) $(3-p)(p+1) < 0$

$\begin{array}{c} (-) \quad (+) \quad (-) \\ -1 \quad 3 \end{array}$ (5)

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Then,



$\therefore p > 3$ (5)

c). $ax^2 + bx + c = 0$ $\begin{array}{l} \alpha \\ \beta \end{array}$ (5) $\begin{array}{l} \alpha + \beta = -b/a \\ \alpha\beta = c/a \end{array}$

$cx^2 - 2bx + 4a = 0$ $\begin{array}{l} \lambda \\ \mu \end{array}$

$x = \frac{2b \pm \sqrt{4b^2 - 4 \cdot c \cdot 4a}}{2c} = \frac{b \pm \sqrt{b^2 - 4ac}}{c}$ (5)

$$x = \frac{b}{c} \pm \sqrt{\frac{b^2}{c^2} - 4\frac{a}{c}}$$

$$= -\left(\frac{\alpha+\beta}{\alpha\beta}\right) \pm \sqrt{\left(\frac{\alpha+\beta}{\alpha\beta}\right)^2 - \frac{4}{\alpha\beta}} \quad (10)$$

$$= -\left(\frac{\alpha+\beta}{\alpha\beta}\right) \pm \sqrt{\frac{(\alpha+\beta)^2\alpha - 4\alpha\beta}{\alpha\beta}} \quad (5)$$

$$= -\left(\frac{\alpha+\beta}{\alpha\beta}\right) \pm \left(\frac{\alpha-\beta}{\alpha\beta}\right)$$

$$\therefore \lambda = -\left(\frac{\alpha+\beta}{\alpha\beta}\right) + \left(\frac{\alpha-\beta}{\alpha\beta}\right) \quad (5)$$

$$\lambda = \underline{\underline{\frac{-2}{\alpha}}} \quad (5)$$

$$\mu = \underline{\underline{\frac{-2}{\beta}}} \quad (5)$$

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$$d). \frac{x^2-1}{x^2(2x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(2x+1)} \quad (10)$$

$$x^2-1 = Ax(2x+1) + B(2x+1) + Cx^2$$

$$\underline{x = -\frac{1}{2}} \quad \frac{1}{4} - 1 = \frac{C}{4} \quad \underline{x=0} \quad -1 = B \quad (5)$$

$$(5) \quad C = -3$$

$$\underline{x=1} \quad 0 = A(3) - 1(3) - 3 \quad A = 2$$

$$\therefore \underline{\underline{\frac{x^2-1}{x^2(2x+1)} = \frac{2}{x} - \frac{1}{x^2} - \frac{3}{(2x+1)}}} \quad (5)$$

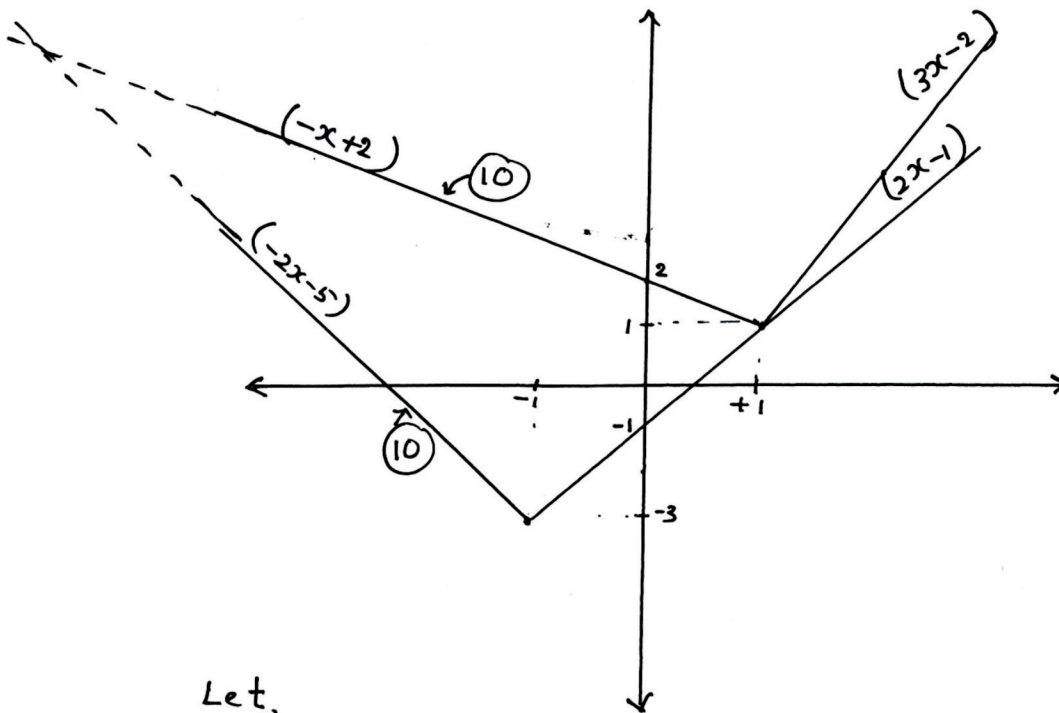
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12) a). $y = 2|x+1| - 3$

$y = x + 2|x-1|$

$$y = \begin{cases} 2(x+1) - 3 & ; x \geq -1 \\ 2x - 1 \\ -2(x+1) - 3 & ; x < -1 \\ -2x - 5 \end{cases} \quad (5)$$

$$y = \begin{cases} x + 2(x-1) & ; x \geq 1 \\ 3x - 2 \\ x - 2(x-1) & ; x < 1 \\ -x + 2 \end{cases} \quad (5)$$



Let,

$$x + 2|x-1| = 2|x+1| - 3;$$

$$\underline{x = 1} \quad (5)$$

$$-x + 2 = -2x - 5$$

$$\underline{x = -7} \quad (5)$$

Let,

$$x + 2|x-1| > 2|x+1| - 3;$$

$$(5) \quad \underline{x > -7} ; x \neq 1 \quad (5)$$

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$$b). \quad a = \log_{2n} n \quad b = \log_{3n} 2n \quad c = \log_{4n} 3n$$

considering;

$$\begin{aligned}
 1+abc &= 1 + \log_{2n} n \cdot \log_{3n} 2n \cdot \log_{4n} 3n \\
 &= 1 + \frac{\log n}{\cancel{\log 2n}} \times \frac{\cancel{\log 2n}}{\cancel{\log 3n}} \times \frac{\cancel{\log 3n}}{\log 4n} \quad (10) \\
 &= 1 + \frac{\log n}{\log 4n} \quad (5) \\
 &= 1 + \log_{4n} n \\
 &= \log_{4n} 4n + \log_{4n} n \quad (5) \\
 &= \log_{4n} 4n^2 = \log_{4n} (2n)^2 \quad (5) \\
 &= 2 \log_{4n} 2n \quad (5) \\
 &= 2 \log_{3n} 2n \times \log_{4n} 3n \quad (5)
 \end{aligned}$$

$$\underline{\underline{1+abc = 2bc}} \quad (5)$$

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12)

$$e) a^x = b^y = c^z = d^w = t \quad (5) + (5)$$

$$x \left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w} \right) = \log_a bcd$$

Considering;

$$x \left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w} \right)$$

$$= \log_a t \left(\frac{1}{\log_b t} + \frac{1}{\log_c t} + \frac{1}{\log_d t} \right) \quad (10)$$

$$= \log_a t \log_t (bcd) \quad (10)$$

$$= \frac{\log_a t \log_a bcd}{\log_a t} \quad (10)$$

$$= \log_a bcd$$

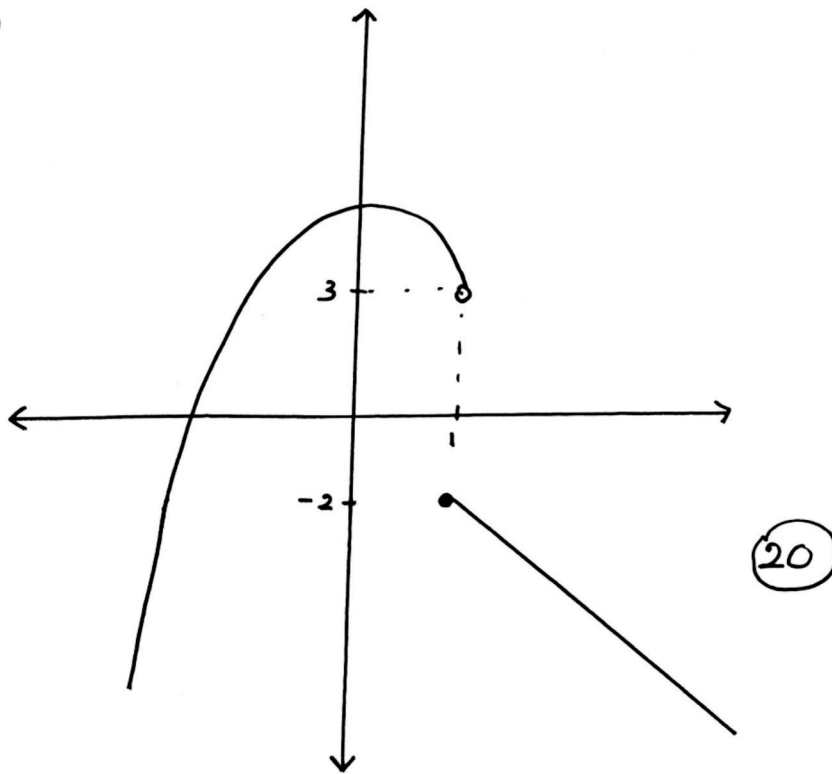
$$\therefore x \left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w} \right) = \log_a bcd \quad (5)$$

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13) $f: \mathbb{R} \rightarrow \mathbb{R}$

$$f(x) = \begin{cases} -x^2 + 4 & ; x < 1 \\ -2x & ; x \geq 1 \end{cases}$$

i)



ii) $\lim_{x \rightarrow 1^-} f(x) = 3$ $\lim_{x \rightarrow 1^+} f(x) = -2$
 (5) (5)

iii) $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$ (10)

Therefore, not continuous at the point $x=1$. (5)

iv) $\lim_{x \rightarrow 1} f(x)$, does not exist. (10)

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b). $\lim_{a \rightarrow 0} \frac{\sin a}{a} = 1$

To proof

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1). $\lim_{x \rightarrow 3} \frac{\sqrt{2x-1} - \sqrt{5}}{\sin(x-3)}$

$= \lim_{x \rightarrow 3} \frac{(2x-1-5)}{\sin(x-3) (\sqrt{2x-1} + \sqrt{5})}$ (5)

$= \lim_{x \rightarrow 3} \frac{2(x-3)}{\sin(x-3) (\sqrt{2x-1} + \sqrt{5})}$

$= 2 \lim_{x \rightarrow 3} \frac{(x-3)}{\sin(x-3)} \lim_{x \rightarrow 3} \frac{1}{(\sqrt{2x-1} + \sqrt{5})}$ (5)

$= 2 \times 1 \times \frac{1}{2\sqrt{5}}$ (5)

$= \frac{1}{\sqrt{5}}$ (5)

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$$\text{ii). } \lim_{x \rightarrow 0} \frac{(\sqrt{4+x^2} - 2)(1 - \cos 2x)}{x^4}$$

$$= \lim_{x \rightarrow 0} \frac{(5)(4+x^2-2)}{(\sqrt{4+x^2}+2)(5)x^4} \frac{2\sin^2 x}{(5)x^4}$$

$$= 2 \lim_{x \rightarrow 0} \frac{(\sin^2 x)}{(5)x^2} \lim_{x \rightarrow 0} \frac{1}{(5)(\sqrt{4+x^2}+2)}$$

$$= 2 \times (1)^2 \times \frac{1}{(5)4}$$

$$= \frac{1}{2} (5)$$

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14). (a). $ax^2 + bx + c = 0$ $\begin{matrix} \alpha \\ \beta \end{matrix}$
For positive real roots,

$$\Delta_x = b^2 - 4ac \geq 0$$

$$\begin{aligned} \alpha + \beta &> 0 \\ \alpha\beta &> 0 \end{aligned} \quad (15)$$

$$ax^2 + a(3b-2c)x + (2b-c)(b-c) + ac = 0$$

$$\Delta_x = b^2 - 4ac \geq 0, \text{ --- (1) } (5)$$

$$\alpha + \beta = -\frac{b}{a} > 0 \text{ --- (2) } \alpha\beta = \frac{c}{a} > 0 \text{ --- (3) } (5)$$

$$a^2x^2 + a(3b-2c)x + (2b-c)(b-c) + ac = 0 \quad \lambda < \mu \quad \text{--- (A)}$$

$$\begin{aligned} \Delta_x &= a^2(3b-2c)^2 - 4a^2\{(2b-c)(b-c) + ac\} \quad (10) \\ &= a^2\{9b^2 - 12bc + 4c^2 - 4(2b^2 - 3bc + c^2 + ac)\} \\ &= a^2\{b^2 - 4ac\} \geq 0 \quad (5) \quad (\text{From (1)}) \end{aligned}$$

$$\begin{aligned} \lambda + \mu &= \frac{a(2c-3b)}{a^2} \\ &= \frac{(2c-3b)}{a} \quad (10) \\ &= 2\left(\frac{c}{a}\right) - 3\left(\frac{b}{a}\right) > 0 \quad ((2) \text{ and } (3)) \end{aligned}$$

$$\begin{aligned} \lambda \mu &= \frac{(2b-c)(b-c) + ac}{a^2} \quad (5) \\ &= \frac{2b^2 - 3bc + c^2 + ac}{a^2} \quad (5) \\ &= 2\left(\frac{b}{a}\right)^2 - 3\left(\frac{b}{a}\right)\left(\frac{c}{a}\right) + \left(\frac{c}{a}\right)^2 + \left(\frac{c}{a}\right) > 0 \\ &\quad \quad \quad (-) \quad (+) \quad \quad \quad (+) \quad (5) \end{aligned}$$

Therefore, eqⁿ (A) has positive (5) real roots. 75

$$\text{Let, } \frac{1}{\lambda} = y \quad (5)$$

λ , root of (A),

$$\text{Then, } \frac{1}{x} = y \Rightarrow x = \frac{1}{y} \quad (5)$$

substituting, $x = \frac{1}{y}$ into (A) $\Rightarrow$

$$a^2 \left(\frac{1}{y}\right)^2 + a(3b-2c)\left(\frac{1}{y}\right) + (2b-c)(b-c) + ac = 0$$

$$\left[(2b-c)(b-c) + ac\right]y^2 + a(3b-2c)y + a^2 = 0 \quad (5)$$

$$\underline{\underline{(2b^2 - 3bc + c^2 + ac)y^2 + a(3b-2c)y + a^2 = 0}} \quad (5) \quad \triangle 20$$

b). (i) A(k, 2) and B(3, 4)

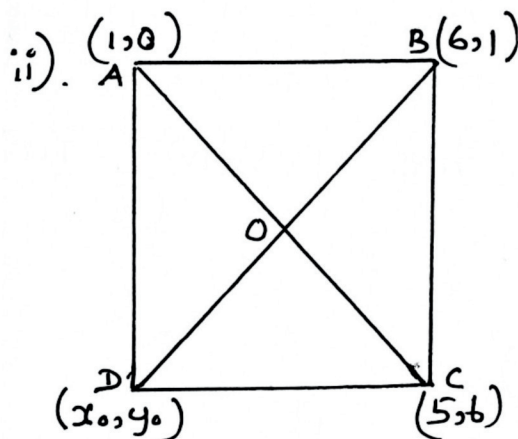
$$AB = (k-3)^2 + (4-2)^2 = 64$$

$$(k-3)^2 + 4 = 64$$

$$(k-3)^2 = 60$$

$$k-3 = \pm 2\sqrt{15}$$

$$\underline{\underline{k = 3 \pm 2\sqrt{15}}} \quad (15)$$



$$O \equiv (3, 3)$$

$$3 = \frac{x_0 + 6}{2}, \quad 3 = \frac{y_0 + 1}{2}$$

$$x_0 = 0, \quad y_0 = 5$$

$$\therefore \underline{\underline{D \equiv (0, 5)}} \quad (15)$$

$$\frac{4x^2}{(4x^2-1)} = \frac{(2x)^2}{(2x-1)(2x+1)}$$

$$x \rightarrow 2x$$

$$a \rightarrow 1 \quad (5)$$

$$b \rightarrow -1$$

$$\frac{4x^2}{(4x^2-1)} = 1 + \frac{1}{2(2x-1)} + \frac{1}{(-2)(2x+1)} \quad (10)$$

$$\frac{4x^2}{(4x^2-1)} = 1 + \frac{1}{2(2x-1)} - \frac{1}{2(2x+1)} \quad (5) \quad \triangle 50$$

$$e). \lim_{x \rightarrow \infty} (\sqrt{x^2+ax+a^2} - \sqrt{x^2+a^2})$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2+ax+a^2} - \sqrt{x^2+a^2}) \times (\sqrt{x^2+ax+a^2} + \sqrt{x^2+a^2})}{(\sqrt{x^2+ax+a^2} + \sqrt{x^2+a^2})} \quad (5)$$

$$= \lim_{x \rightarrow \infty} \frac{x^2+ax+a^2-x^2-a^2}{\sqrt{x^2+ax+a^2} + \sqrt{x^2+a^2}} \quad (5)$$

$$= \lim_{x \rightarrow \infty} \frac{ax}{(\sqrt{x^2+ax+a^2} + \sqrt{x^2+a^2})} \quad (5)$$

$$= \lim_{x \rightarrow \infty} \frac{a}{(\sqrt{1+\frac{a}{x}+\frac{a^2}{x^2}} + \sqrt{1+\frac{a^2}{x^2}})} = \frac{a}{\sqrt{1}+\sqrt{1}} \quad (5)$$

$$(5) = \frac{a}{2}$$

$$\triangle 30$$

(16) a). $\sin(A+B) = \sin A \cos B + \cos A \sin B$ (5)

$$\sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \sin \frac{\pi}{6} \cos \frac{\pi}{4} + \cos \frac{\pi}{6} \sin \frac{\pi}{4}$$

$$\sin\left(\frac{5\pi}{12}\right) = \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \quad (5)$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} \quad (5)$$

$$\underline{\underline{\sin\left(\frac{5\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}}} \quad (5)$$

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Hence,

$$\cos\left(\frac{5\pi}{6}\right) = 1 - 2\sin^2\left(\frac{5\pi}{12}\right) \quad (5)$$

$$\quad (5) = 1 - 2\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)^2$$

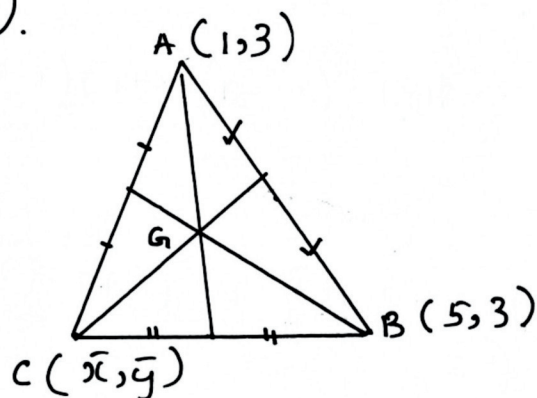
$$= 1 - \frac{1}{8} (8 + 2\sqrt{12}) \quad (5)$$

$$= 1 - 1 - \frac{1}{4} 2\sqrt{3}$$

$$\underline{\underline{\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}}} \quad (5)$$

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iii).



$$G = \left(\frac{1+5+\bar{x}}{3}, \frac{3+3+\bar{y}}{3} \right)$$

$$\frac{10}{3} = \frac{6+\bar{x}}{3}$$

$$4 = \frac{6+\bar{y}}{3}$$

$$\bar{x} = 4$$

$$\bar{y} = 6$$

$$\therefore \underline{\underline{C(4,6)}}$$

(25)



(15) (es). Proof - Factor Theorem



$$f(x) = x^4 + ax^3 + bx + c = (x-1)(x+1)(x-2)(x+\lambda)$$

$$\underline{x=1}$$

$$1+a+b+c = 0$$

$$a+b+c = -1 \quad \text{--- (1)} \quad (5)$$

$$\underline{x=2}$$

$$16+8a+2b+c = 0$$

$$8a+2b+c = -16 \quad \text{--- (2)} \quad (5)$$

$$\underline{x=-1}$$

$$1-a-b+c = 0$$

$$c-a-b = -1 \quad \text{--- (3)}$$

$$\underline{\underline{a = -\frac{5}{2}}}$$

$$\underline{\underline{b = \frac{5}{2}}}$$

$$\underline{\underline{c = -1}}$$

$$\underline{\underline{\lambda = -\frac{1}{2}}} \quad (10)$$

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$$\Rightarrow \underline{\underline{\left(x - \frac{1}{2}\right)}} \quad (5) \quad f(x) = (x^4 - 1)(x+1)(x-2)\left(x - \frac{1}{2}\right)$$

$$2f(x+1) = x^2 + x - 2$$

$$2 \left\{ x(x+2)(x-1) \left(x + \frac{1}{2}\right) \right\} = (x+2)(x-1) \quad (5)$$

$$(x+2)(x-1) \left\{ 2x\left(x + \frac{1}{2}\right) - 1 \right\} = 0 \quad (5)$$

$$(x+2)(x-1)(2x^2 + x - 1) = 0$$

$$(x+2)(x-1)(2x-1)(x+1) = 0 \quad (5)$$

solutions are,

$$\underline{\underline{x = -2}} \quad \underline{\underline{x = 1}} \quad \underline{\underline{x = \frac{1}{2}}} \quad \underline{\underline{x = -1}} \quad (10) \quad \triangle 55$$

b).

$$\frac{x^2}{(x-a)(x-b)} = p + \frac{q}{(x-a)} + \frac{r}{(x-b)} \quad (5)$$

$$x^2 = p(x-a)(x-b) + q(x-b) + r(x-a)$$

$$x^2 \rightarrow 1 = p$$

$$x \rightarrow 0 = -p(a+b) + q + r \quad (10)$$

$$x^0 \rightarrow 0 = abp - bq - ar$$

$$\underline{\underline{p = 1}} \quad (5) \quad \underline{\underline{q = \frac{a^2}{a-b}}} \quad (5) \quad \underline{\underline{r = \frac{b^2}{b-a}}} \quad (5)$$

$$\underline{\underline{\frac{x^2}{(x-a)(x-b)} = 1 + \frac{a^2}{(a-b)(x-a)} + \frac{b^2}{(b-a)(x-b)}}}$$

For Educational Purpose Only

$$d). \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$$

$$\text{Let, } \alpha = \tan^{-1}\left(\frac{1}{2}\right) \quad \beta = \tan^{-1}\left(\frac{1}{3}\right)$$

$$\tan \alpha = \frac{1}{2} \quad \tan \beta = \frac{1}{3} \quad (5)$$

Then,

$$\alpha + \beta = \frac{\pi}{4} \quad (5)$$

Prove that,

$$\tan(\alpha + \beta) = \tan \frac{\pi}{4} = 1 \quad (5)$$

$$\text{L.H.S. } \tan(\alpha + \beta)$$

$$= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \quad (5)$$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}} \quad (5)$$

$$= \frac{5}{5} = 1 \quad (5)$$

$$\therefore \underline{\underline{\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}}}$$



If,

$$b). \alpha + \beta - \gamma = \pi,$$

$$\sin^2 \alpha + \sin^2 \beta - \sin^2 \gamma = 2 \sin \alpha \cos \gamma \sin \beta$$

$$L.H.S \quad \sin^2 \alpha + \sin^2 \beta - \sin^2 \gamma$$

$$= \sin^2 \alpha + (\sin \beta - \sin \gamma)(\sin \beta + \sin \gamma)$$

$$= \sin^2 \alpha + 2 \cos \left(\frac{\beta + \gamma}{2} \right) \sin \left(\frac{\beta - \gamma}{2} \right) 2 \sin \left(\frac{\beta + \gamma}{2} \right) \cos \left(\frac{\beta - \gamma}{2} \right)$$

$$= \sin^2 \alpha + \sin(\beta + \gamma) \sin(\beta - \gamma)$$

$$= \sin^2 \alpha + \sin(\beta + \gamma) \sin \alpha$$

$$= \sin \alpha [\sin \alpha + \sin(\beta + \gamma)]$$

$$= \sin \alpha [\sin(\beta + \gamma) - \sin(\gamma - \beta)]$$

$$= \sin \alpha \cdot 2 \cos \gamma \sin \beta$$

$$= \underline{\underline{2 \sin \alpha \cos \gamma \sin \beta}}$$

45

$$c). 2 \cos^2 x + \sqrt{3} \sin x + 1 = 0$$

$$2(1 - \sin^2 x) + \sqrt{3} \sin x + 1 = 0$$

$$2 \sin^2 x - \sqrt{3} \sin x - 3 = 0$$

$$(2 \sin x + \sqrt{3})(\sin x - \sqrt{3}) = 0$$

$$\sin x = -\frac{\sqrt{3}}{2} \quad \text{or} \quad \sin x = \sqrt{3}$$

30

$$\sin x = \sin \left(-\frac{\pi}{3} \right) \Rightarrow \underline{\underline{x = n\pi + (-1)^n \left(-\frac{\pi}{3} \right); n \in \mathbb{Z}}}$$

5

5

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(17) To state - Sine Rule. (05)

$$a) \frac{a^2+b^2}{a^2+c^2} = \frac{1 + \cos(A-B)\cos C}{1 + \cos(A-C)\cos B}$$

L.H.S.

from sine Rule,

$$= \frac{a^2+b^2}{a^2+c^2}$$

$$(5) \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{\lambda}$$

$$= \frac{\sin^2 A + \sin^2 B}{\sin^2 A + \sin^2 C} (5)$$

$$A+B+C = \pi (5)$$

$$= \frac{1 - \cos 2A + 1 - \cos 2B}{1 - \cos 2A + 1 - \cos 2C} (10)$$

$$= \frac{2 - (\cos 2A + \cos 2B)}{2 - (\cos 2A + \cos 2C)}$$

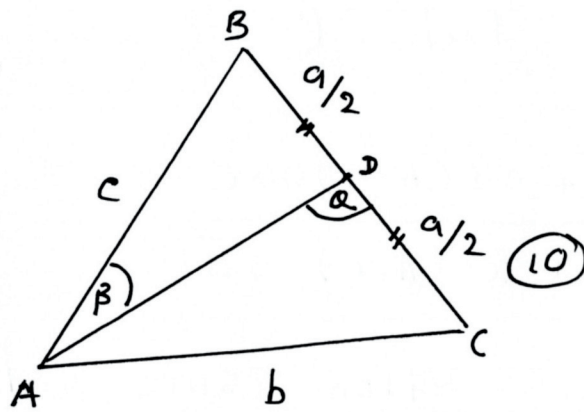
$$= \frac{2 - 2\cos(A+B)\cos(A-B)}{2 - 2\cos(A+C)\cos(A-C)} (10)$$

$$= \frac{1 + \cos(A-B)\cos C}{1 + \cos(A-C)\cos B} (10)$$

$$\therefore \frac{a^2+b^2}{a^2+c^2} = \frac{1 + \cos(A-B)\cos C}{1 + \cos(A-C)\cos B}$$



b).



ABD Δ , cosine Rule,

$$\cos \hat{A}DB = \frac{AD^2 + \left(\frac{a}{2}\right)^2 - c^2}{2AD \cdot \left(\frac{a}{2}\right)} \quad (10) \quad (1)$$

ADC Δ , cosine Rule,

$$\cos \hat{A}DC = \cos(\pi - \hat{A}DB) = -\cos \hat{A}DB \quad (5)$$

$$-\cos \hat{A}DB = \frac{AD^2 + \left(\frac{a}{2}\right)^2 - b^2}{2AD \cdot \left(\frac{a}{2}\right)} \quad (10) \quad (2)$$

From (1) and (2),

$$0 = 2AD^2 + 2\left(\frac{a}{2}\right)^2 - c^2 - b^2 \quad (5)$$

$$2AD^2 = b^2 + c^2 - \frac{a^2}{2}$$

$$AD^2 = \frac{2b^2 + 2c^2 - a^2}{4} \quad (5)$$

$$AD = \frac{\sqrt{2b^2 + 2c^2 - a^2}}{2} \quad (5)$$

If $\hat{B}AD = \beta,$

$$\frac{\sin \beta}{a/2} = \frac{\sin B}{AD} \quad (10)$$

$$\sin \beta = \frac{a \sin B \times 2}{2 \sqrt{2b^2 + 2c^2 - a^2}} \quad (10)$$

$$\therefore \sin \beta = \frac{a \sin B}{\sqrt{2b^2 + 2c^2 - a^2}} \quad (5)$$

25

If, $\hat{ADC} = \alpha,$

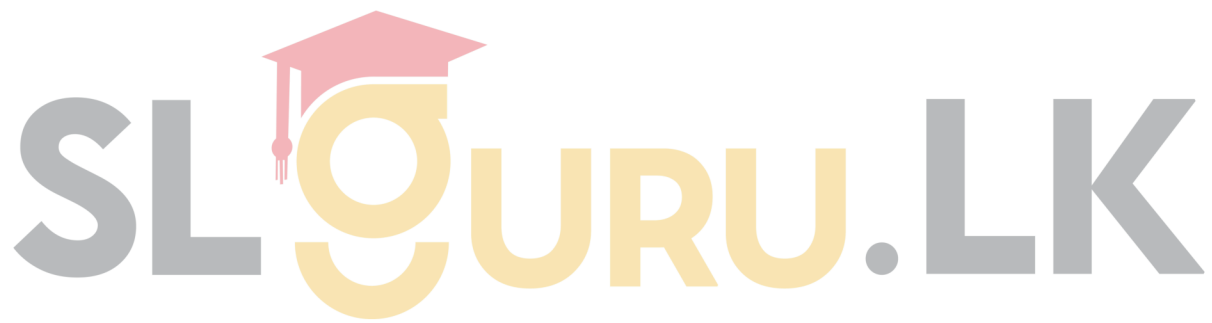
$$\frac{\sin \alpha}{b} = \frac{\sin C}{AD} \quad (10)$$

$$\sin \alpha = \frac{b \sin C \times 2}{2 \sqrt{2b^2 + 2c^2 - a^2}} \quad (10)$$

$$\sin \alpha = \frac{2b \sin C}{\sqrt{2b^2 + 2c^2 - a^2}} \quad (5)$$

25

“Education is the foundation upon which we build our future.” —
Christine Gregoire

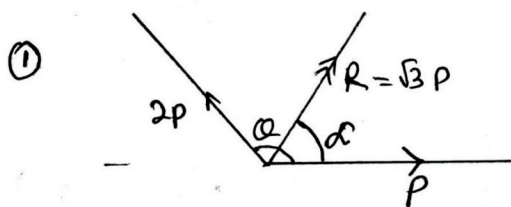


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Second Term Test - 2020
Marking Scheme

COMBINED MATHEMATICS - 11

Grade -12



$$3p^2 = p^2 + 4p^2 + 4p^2 \cos \alpha \quad (5)$$

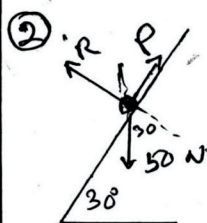
$$4p^2 \cos \alpha = -2p^2$$

$$\cos \alpha = -\frac{1}{2} \quad (5)$$

$$\alpha = \frac{2\pi}{3} \quad (5)$$

$$\tan \alpha = \frac{2p \cos 120}{p + 2p \cos 120} \quad (5)$$

$$\Rightarrow \alpha = \pi/2 \quad (5) \quad \boxed{25}$$



$$\frac{P}{\sin 150^\circ} = \frac{R}{\sin 120^\circ} = \frac{50}{\sin 90^\circ}$$

$$\frac{P}{\frac{1}{2}} = \frac{R}{\frac{\sqrt{3}}{2}} = 50 \Rightarrow P = 25 \text{ N}, R = 25\sqrt{3} \text{ N}.$$

 $\boxed{25}$

③.

$$3a + 5b = 8c$$

$$3a + 5b = 3c + 5c \quad (5)$$

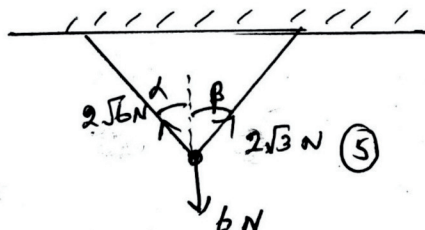
$$3a - 3c = 5c - 5b$$

$$3(a - c) = 5(c - b) \quad (5)$$

$$\therefore 3\vec{CA} = 5\vec{BC} \Rightarrow A, B, C \text{ collinear.} \quad (5)$$

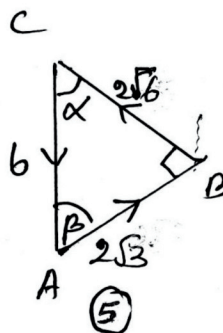
$$\frac{AC}{CB} = \frac{5}{3} \quad (5) \quad \boxed{25}$$

④.



$$(2\sqrt{3})^2 + (2\sqrt{3})^2 = 6^2$$

$\Rightarrow ABC$ is a right angled triangle. (5)



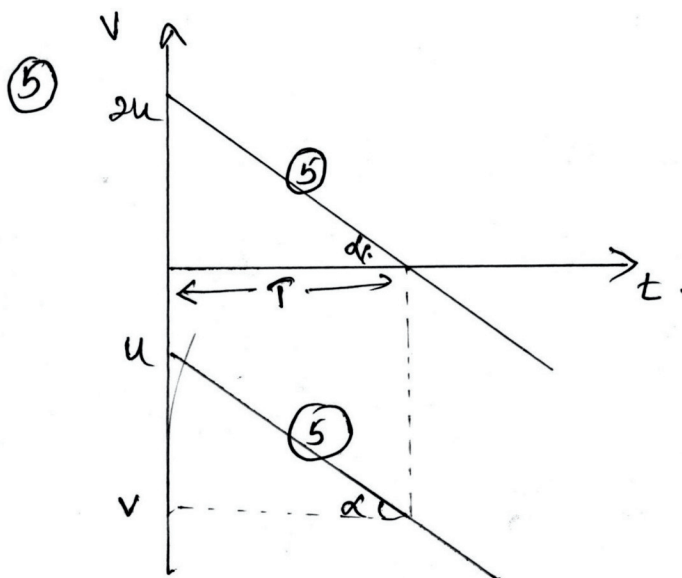
$$\sin \beta = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$$

$$\beta = \sin^{-1}\left(\frac{\sqrt{3}}{3}\right) \quad (5)$$

$$\sin \alpha = \frac{2\sqrt{3}}{6} = \frac{1}{\sqrt{3}}$$

$$\alpha = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \quad (5)$$

 $\boxed{25}$



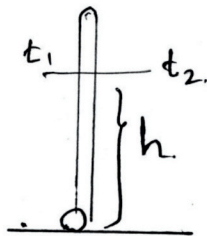
$$\tau \text{ and } \alpha = \frac{2u}{T} = \frac{v-u}{T}$$

$$2u = v - u \quad (5)$$

$$\underline{v = 3u} \quad (5)$$

25

⑥.



$$s = ut + \frac{1}{2}at^2$$

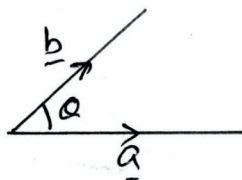
$$h = ut - \frac{1}{2}gt^2 \quad (5)$$

$$gt^2 = 2ut - 2h$$

$$gt^2 - 2ut + 2h = 0 \quad (5)$$

$$\begin{aligned} & \left. \begin{array}{l} t_1 \\ t_2 \end{array} \right\} \begin{array}{l} t_1, t_2 = \frac{2h}{g} \\ h = \frac{gt_1 t_2}{2} \end{array} \quad (10) \\ & \underline{25} \quad (5) \end{aligned}$$

⑦.



$$|a| = 1$$

$$|b| = 1$$

$$a \cdot b = |a||b|\cos\theta$$

$$a \cdot b = \cos\theta \quad (5)$$

$$|a-b|^2 = (a-b) \cdot (a-b) \quad (5)$$

$$= a^2 + b^2 - 2a \cdot b$$

$$= 1 + 1 - 2a \cdot b$$

$$= 2 - 2(\cos\theta) \quad (5)$$

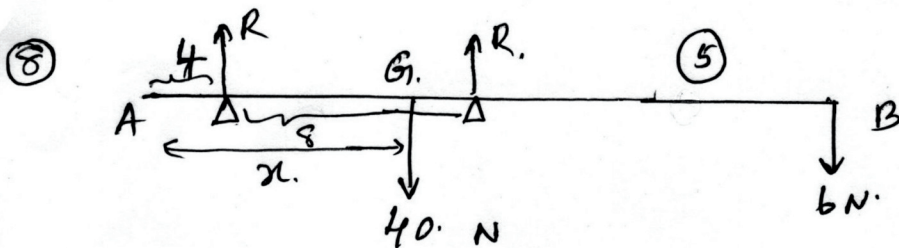
$$= 2 - 2[1 - 2\sin^2\theta/2] \quad (5)$$

$$= 4\sin^2\theta/2$$

$$\Rightarrow |a-b| = 2\sin\theta/2 \quad (5)$$

$$\sin\theta/2 = \frac{1}{2}|a-b|$$

25



$$\uparrow 2R - 40 - 6 = 0 \quad (5)$$

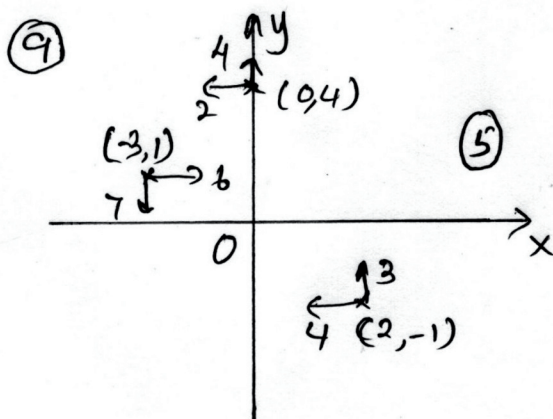
$$R = 23 \text{ N.} \quad (5)$$

$$\curvearrowleft A) 4 \times R + 8 \times R - 40 \times x - 24 \times 6 = 0 \quad (5)$$

$$16R - 144 - 40x = 0.$$

$$224 - 144 - 40x = 0$$

$$x = \frac{28}{5} \text{ m.} \quad (5) \quad \boxed{25}$$

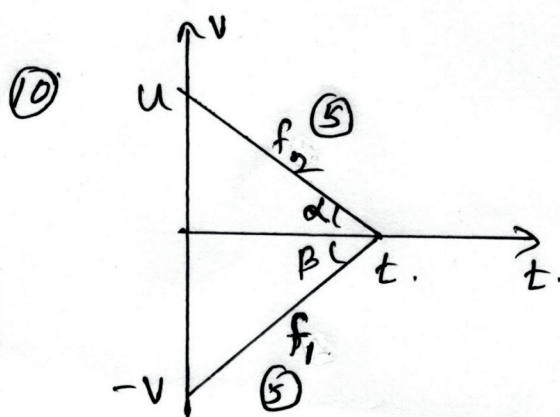


$$\rightarrow x = 6 - 4 - 2 = 0 \quad (5)$$

$$\uparrow y = 4 + 3 - 7 = 0 \quad (5)$$

$$\curvearrowleft G_{\text{cc}} = 3 \times 2 + 2 \times 4 - 6 \times 1 + 7 \times 3 - 4 \times 1 \quad (5)$$

$$= 25 \text{ units.} \quad (5) \quad \boxed{25}$$



$$\tan \alpha = \frac{u}{t} = f_2 \quad \tan \beta = \frac{v}{t} = f_1$$

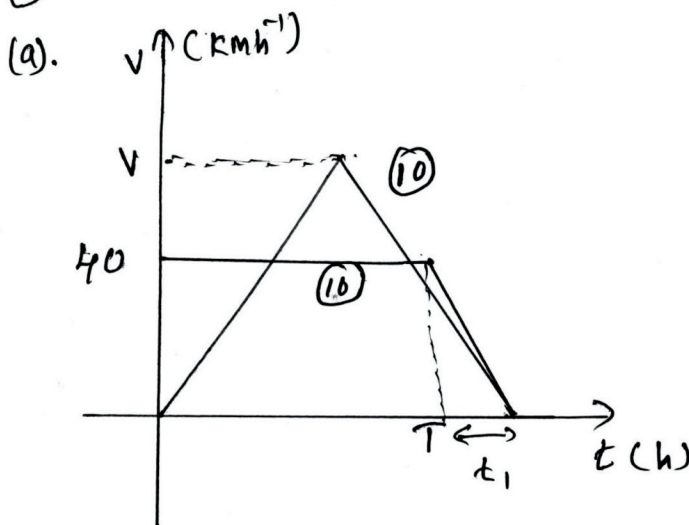
$$d = \frac{1}{2} t \times u + \frac{1}{2} t \times v \quad (5)$$

$$= \frac{1}{2} \frac{u}{f_2} \cdot u + \frac{1}{2} \left(\frac{v}{f_1} \right) v \quad (5)$$

$$= \frac{1}{2} \frac{u^2}{f_2} + \frac{1}{2} \frac{v^2}{f_1}$$

$$d = \frac{u^2}{2f_2} + \frac{v^2}{2f_1} \quad (5) \quad \boxed{25}$$

(11)



$$40 \times T = 7 \quad (10)$$

$$T = \frac{7}{40} \text{ hours.}$$

$$\frac{1}{2} \times 40 \times t_1 = \frac{3}{2} \quad (16)$$

$$t_1 = \frac{3}{40} \text{ hours.}$$

$$\therefore \text{Total time} = \frac{7}{40} + \frac{3}{40} \quad (5)$$

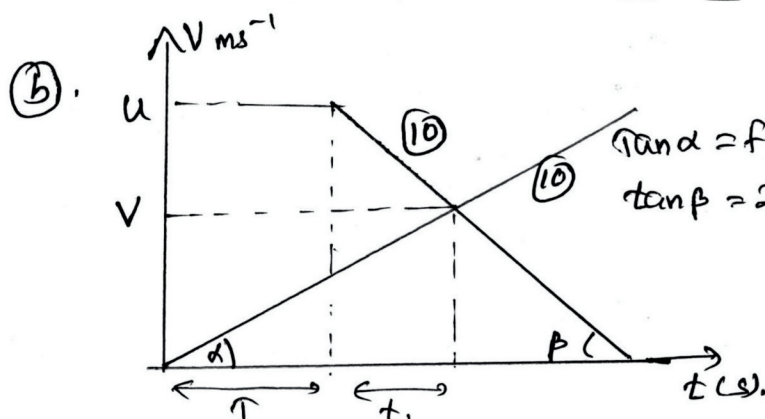
$$= \frac{1}{4} \text{ hours}$$

$$= 25 \text{ minutes.}$$

$$(ii) \quad \frac{1}{2} \times V \times \frac{1}{4} = 8.5 \quad (5)$$

$$V = 68 \text{ kmh}^{-1} \quad (5)$$

60



$$\tan \alpha = f \quad (5)$$

$$\tan \beta = 2f \quad (5)$$

* If vehicles just meet, their displacements are equal when their velocities are equal.

$$\tan \alpha = \frac{V}{T+t}$$

$$f = \frac{V}{T+t} \Rightarrow V = f(T+t) \quad (2) \quad (10)$$

$$\tan \beta = \frac{u-V}{t} = 2f \Rightarrow V = u - 2ft \quad (3)$$

$$(2) = (3)$$

$$f(T+t) = u - 2ft$$

$$t = \left(\frac{u - fT}{3f} \right) \quad (10)$$

$$\text{from (3)} \Rightarrow V = u - 2f \left(\frac{u - fT}{3f} \right)$$

$$V = \frac{fu + 2f^2T}{3f} \quad (10)$$

from ① $ut = v\tau$

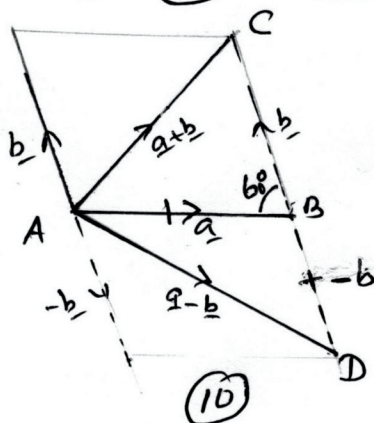
$$u\left(\frac{u-f\tau}{3f}\right) = \left[\frac{fu+2f^2\tau}{3f}\right]\tau \quad (5)$$

$$u^2 - f\tau u = f\tau u + 2f^2\tau^2$$

$$\therefore \underline{u^2 = 2f\tau [u + f\tau]} \quad (5)$$

90

⑫ (a). Theory - (10) 10



$$|a| = |b| = |a+b| = 1$$

$\Rightarrow$ ABC is an equilateral Δ . (5)

$$\therefore \hat{ABC} = 60^\circ \quad (5)$$

$$\therefore \hat{ADC} = 30^\circ$$

$\therefore$ from ΔABD :

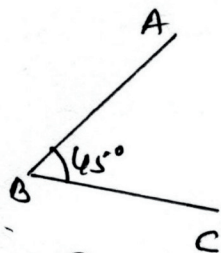
$$\frac{AB}{\sin 30^\circ} = \frac{AD}{\sin 120^\circ} \quad (5)$$

$$AD = \sqrt{3}$$

$$\underline{|a-b| = \sqrt{3}} \quad (5)$$

30

$$\left. \begin{aligned} \vec{OA} &= 4\hat{i} + 2\hat{j} \\ \vec{OB} &= \hat{i} + \hat{j} \\ \vec{OC} &= (k+1)\hat{i} + 6\hat{j} \end{aligned} \right\} \quad (5)$$



$$\vec{AB} = \underline{b} - \underline{a} = -3\hat{i} - \hat{j}$$

$$\vec{BC} = \underline{c} - \underline{a} = k\hat{i} + 5\hat{j} \quad (5) \text{ for both.}$$

$$\left. \begin{aligned} |\vec{AB}| &= \sqrt{10} \\ |\vec{BC}| &= \sqrt{k^2 + 25} \end{aligned} \right\} \quad (5)$$

$$\vec{AB} \cdot \vec{BC} = |\vec{AB}| |\vec{BC}| \cos 45^\circ \quad (5)$$

$$(-3\hat{i} - \hat{j}) \cdot (k\hat{i} + 5\hat{j}) = \sqrt{10} \sqrt{k^2 + 25} \times \frac{1}{\sqrt{2}}$$

$$(-3k - 5) = \sqrt{5} \sqrt{k^2 + 25} \quad (5)$$

$$9k^2 + 25 + 30k = 5k^2 + 125$$

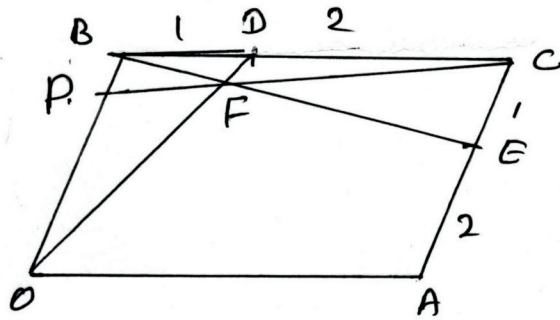
$$2k^2 + 15k - 50 = 0 \quad (5)$$

$$(2k-5)(k+10) = 0$$

$$k = 5/2 \quad (5) \quad k = -10 \quad \#$$

35

(c).



$$\begin{aligned}\vec{OA} &= \underline{a} \\ \vec{OB} &= \underline{b} \quad (5)\end{aligned}$$

$$\begin{aligned}\vec{OF} &= \lambda \vec{OD} \\ &= \lambda [\vec{OB} + \vec{BD}] \quad (5) \\ &= \lambda [\vec{OB} + \frac{1}{3} \vec{BC}] \\ \vec{OF} &= \lambda [\underline{b} + \frac{1}{3} \underline{a}] \quad (5)\end{aligned}$$

$$\begin{aligned}\vec{OF} &= \vec{OB} + \vec{BF} \\ &= \underline{b} + \mu \vec{BE} \quad (5) \\ &= \underline{b} + \mu [\vec{BC} + \vec{CE}] \\ &= \underline{b} + \mu [\underline{a} + \frac{1}{3} \vec{CA}] \quad (5) \\ \vec{OF} &= \underline{b} + \mu [\underline{a} - \frac{1}{3} \underline{b}] \quad (5)\end{aligned}$$

$$\Rightarrow \lambda [\underline{b} + \frac{1}{3} \underline{a}] = \underline{b} + \mu [\underline{a} - \frac{1}{3} \underline{b}] \quad (5)$$

$$\lambda = 1 - \frac{1}{3} \mu \quad (5)$$

$$\frac{\lambda}{3} = \mu \quad (5)$$

$$\therefore \lambda = 3\mu$$

$$\frac{10\mu}{3} = 1$$

$$\mu = \frac{3}{10} \Rightarrow \lambda = \frac{9}{10} \quad (5)$$

$$\begin{aligned}\therefore \vec{OF} &= \frac{9}{10} (\underline{b} + 3\underline{a}) \quad (5) \\ &= \underline{\underline{\frac{3}{10} (3\underline{a} + \underline{b})}}\end{aligned}$$

Let;

$$OP:PB = 1:k$$

$$\therefore \vec{OP} = \frac{1}{1+k} \underline{b} \quad (5)$$

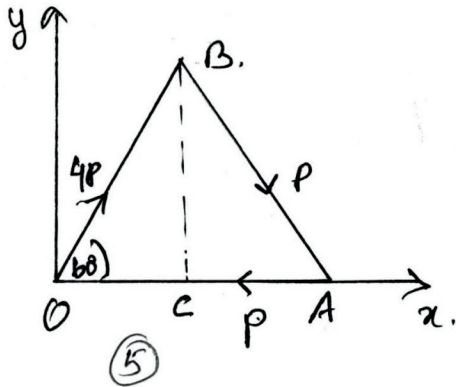
$$\begin{aligned}\vec{OP} &= \vec{OF} + \vec{FP} \\ &= \vec{OF} + \gamma \vec{CP} \quad (5) \\ &= \vec{OF} + \gamma [\vec{CB} + \vec{BP}]\end{aligned}$$

$$\vec{OP} = \frac{3}{10} (3\underline{a} + \underline{b}) + \gamma \left[-\underline{a} + \frac{k}{1+k} (-\underline{b}) \right] \quad (5)$$

$$\therefore \frac{1}{1+k} \underline{b} = \frac{3}{10} (3\underline{a} + \underline{b}) + \gamma \left[-\underline{a} + \frac{k}{1+k} (-\underline{b}) \right] \Rightarrow \gamma = \frac{3}{10}; k = \frac{1}{6} \quad (5)$$

$$\therefore OP:PB = 1:6 \quad (5)$$

(13)



$$4p \Rightarrow 4p \cos 60^\circ \hat{i} + 4p \sin 60^\circ \hat{j} \\ = 2p \hat{i} + 2\sqrt{3}p \hat{j} \quad (5)$$

$$p \Rightarrow p \cos 60^\circ \hat{i} - p \cos 30^\circ \hat{j} \\ = \frac{p}{2} \hat{i} - \frac{\sqrt{3}}{2} p \hat{j} \quad (5)$$

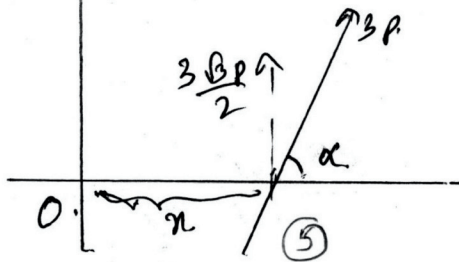
$$p \Rightarrow -p \hat{i} + 0 \hat{j} \quad (5)$$

$$R = (2p + \frac{p}{2} - p) \hat{i} + (2\sqrt{3}p - \frac{\sqrt{3}}{2}p) \hat{j} \quad (5)$$

$$R = (\frac{3p}{2}) \hat{i} + (\frac{3\sqrt{3}}{2}p) \hat{j} \quad (5)$$

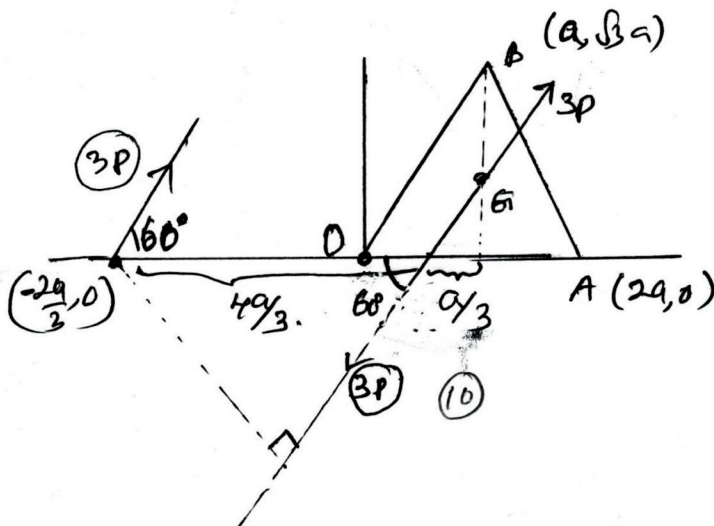
$$R = \sqrt{\frac{9p^2}{4} + \frac{27p^2}{4}} = 3p \quad (5) \quad \tan \alpha = \frac{3\sqrt{3}p}{\frac{3p}{2}} \quad (5)$$

$$\alpha = 60^\circ$$



(5)

$$\frac{3\sqrt{3}p}{2} \times \alpha = -p \sin 60^\circ \times 2a \\ \alpha = -\frac{2a}{3} \quad (5)$$



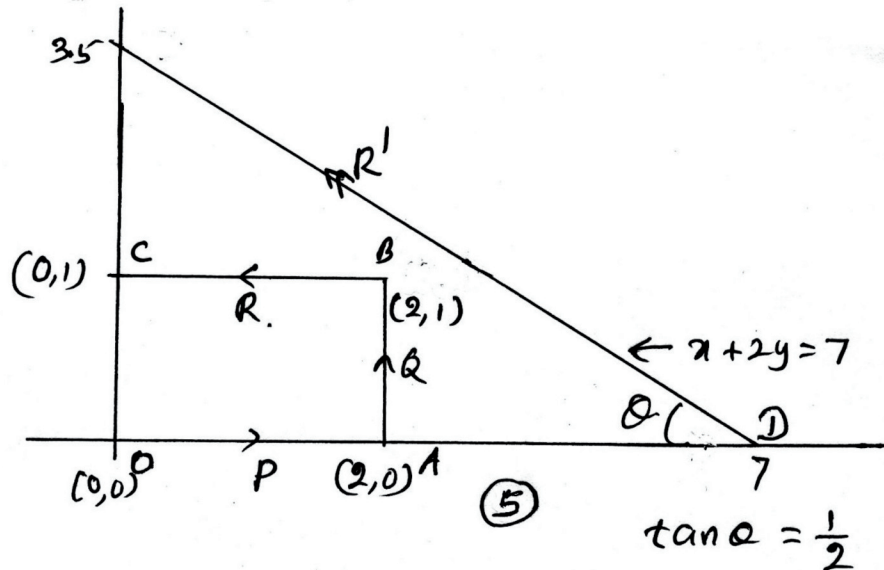
$$G = [a, \frac{a}{\sqrt{3}}]$$

$$M = 3p \times \frac{4a}{3} \sin 60^\circ = \quad (10)$$

$$= 3p \times \frac{4a}{3} \times \frac{\sqrt{3}}{2} = \underline{\underline{2\sqrt{3}pa}} \quad (10)$$

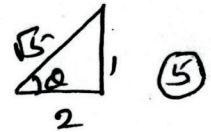
90

(13) (b).



$$\rightarrow x \Rightarrow P - R = -R' \cos \theta \quad \text{--- (1) (5)}$$

$$\uparrow y \Rightarrow Q = R' \sin \theta \quad \text{--- (2) (5)}$$



$$\textcircled{1}) \quad R \times 1 - Q \times 5 = 0.$$

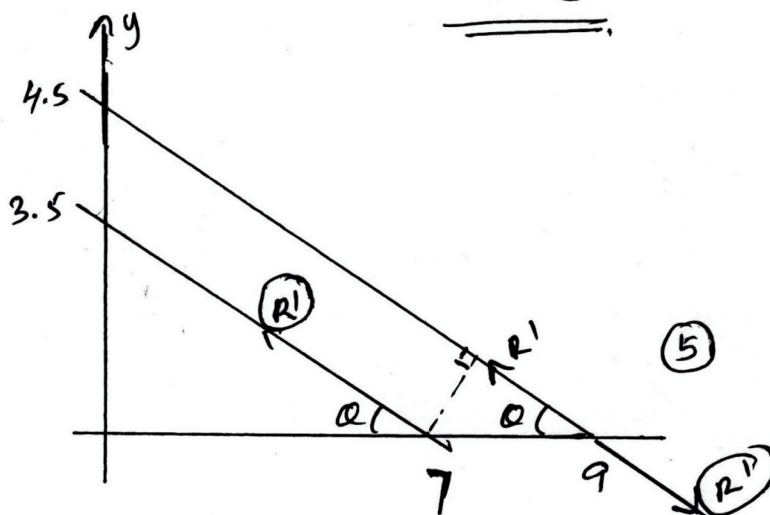
$$R = 5Q \quad \text{--- (3) (5)}$$

$$\text{From (1), (2), (3)} \quad P - 5R' \sin \theta = -R' \cos \theta \quad \text{(5)}$$

$$P = R' [5 \sin \theta - \cos \theta]$$

$$= R' \left[5 \times \frac{1}{\sqrt{5}} - \frac{2}{\sqrt{5}} \right] \quad \text{(5)}$$

$$\underline{\underline{R' = \frac{\sqrt{5}P}{3} \quad \text{(5)}}}$$



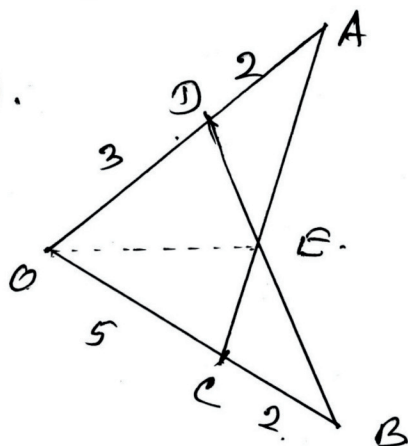
$$G = R' \times 2 \sin \theta \quad \text{(5)}$$

$$= \frac{\sqrt{5}P}{3} \times 2 \times \frac{1}{\sqrt{5}} = \frac{2P}{3}$$

$\therefore$ couple should be applied is

$$G = \frac{2P}{3} \text{ units.} \quad \text{(5) (60)}$$

(14).



$$\vec{OA} = \underline{a} \quad (5)$$

$$\vec{OB} = \underline{b}$$

$$\begin{aligned} \vec{OE} &= \vec{OB} + \vec{BE} \quad (5) \\ &= \vec{OB} + \lambda \vec{BD} \quad (5) \\ &= \vec{OB} + \lambda [\vec{BO} + \vec{OD}] \\ &= \vec{OB} + \lambda [\vec{BO} + \frac{3}{5}\vec{OA}] \quad (5) \\ \vec{OE} &= \underline{b} + \lambda [\frac{3}{5}\underline{a} - \underline{b}] \quad (5) \end{aligned}$$

$$\begin{aligned} \vec{OE} &= \vec{OA} + \vec{AE} \quad (5) \\ &= \vec{OA} + \mu \vec{AC} \quad (5) \\ &= \vec{OA} + \mu (\vec{AO} + \vec{OC}) \quad (5) \\ \vec{OE} &= \underline{a} + \mu [\frac{5}{7}\underline{b} - \underline{a}] \quad (5) \end{aligned}$$

$$\therefore \underline{a} + \mu [\frac{5}{7}\underline{b} - \underline{a}] = \underline{b} + \lambda [\frac{3}{5}\underline{a} - \underline{b}] \quad (10)$$

$$\therefore 1 - \mu = \frac{3}{5}\lambda \quad \text{--- (A)} \quad \Rightarrow 7\lambda + 5\mu = 7 \quad \text{--- (15)}$$

$$1 - \lambda = \frac{5}{7}\mu \quad \text{--- (B)} \quad \Rightarrow 3\lambda + 5\mu = 5 \quad \text{--- (20)}$$

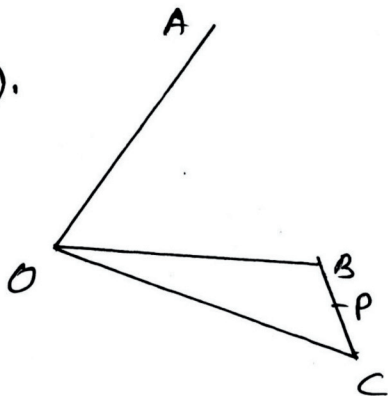
$$\text{from (1) and (2)} \quad \lambda = \frac{1}{2} \quad (5)$$

$$\therefore \vec{OE} = \underline{b} + \frac{1}{2} [\frac{3}{5}\underline{a} - \underline{b}]$$

$$\vec{OE} = \underline{\underline{\left[\frac{b}{2} + \frac{3}{10}a \right]}} \quad (5)$$

75

(15).



$$\begin{aligned} \vec{OA} &= \underline{a} \\ \vec{OB} &= \underline{b} \quad (5) \\ \vec{OC} &= \underline{c} \end{aligned}$$

$$\begin{aligned} \vec{OP} &= \vec{OC} + \vec{CP} \quad (5) \\ &= \underline{c} - \frac{1}{10}\vec{BC} \\ &= \underline{c} - \frac{1}{10}(\underline{c} - \underline{b}) \quad (5) \end{aligned}$$

$$\vec{OP} = \underline{\underline{\frac{1}{10}(9c + b)}} \quad (5) \quad 20$$

$$(ii). A p \perp BC. \Rightarrow \vec{AP} \cdot \vec{BC} = 0 \quad (5)$$

$$\therefore (\vec{AO} + \vec{OP}) \cdot (\vec{BO} + \vec{OC}) = 0 \quad (10)$$

$$(-\underline{a} + \underline{p}) \cdot (\underline{c} - \underline{b}) = 0 \quad (5)$$

$$\underline{p} \cdot (\underline{c} - \underline{b}) - \underline{a} \cdot (\underline{c} - \underline{b}) = 0.$$

$$\frac{1}{10} (\underline{ac} + \underline{b}) \cdot (\underline{c} - \underline{b}) = \underline{a} \cdot (\underline{c} - \underline{b})$$

$$(\underline{ac} + \underline{b}) \cdot (\underline{c} - \underline{b}) = 10 \underline{a} \cdot (\underline{c} - \underline{b}). \quad \text{--- (1)} \quad (30)$$

(b). If OA, OB and OC are perpendicular to each other;

$$\vec{OA} \cdot \vec{OB} = 0$$

$$\vec{OB} \cdot \vec{OC} = 0$$

$$\vec{OA} \cdot \vec{OC} = 0$$

$$\underline{a} \cdot \underline{b} = 0 \quad (5)$$

$$\underline{b} \cdot \underline{c} = 0 \quad (5)$$

$$\underline{a} \cdot \underline{c} = 0 \quad (5)$$

$$\text{From (1): } 10 (\underline{a} \cdot \underline{c} - \underline{a} \cdot \underline{b}) = (\underline{ac} + \underline{b}) \cdot (\underline{c} - \underline{b})$$

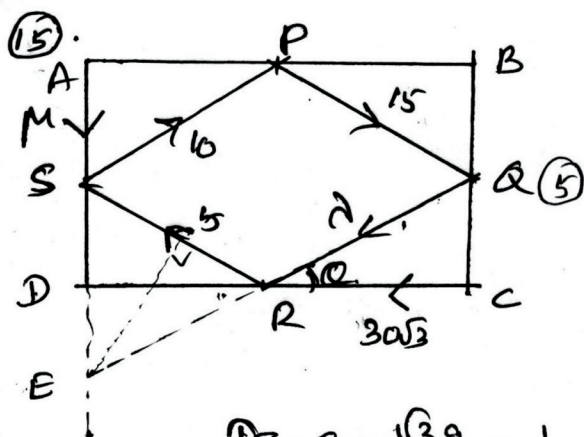
$$(5) 0 = (\underline{ac} + \underline{b}) \cdot (\underline{c} - \underline{b})$$

$$\therefore \underline{ac} \cdot \underline{c} - \underline{ac} \cdot \underline{b} + \underline{b} \cdot \underline{c} - \underline{b} \cdot \underline{b} = 0$$

$$\underline{ac} \cdot \underline{c} - \underline{b} \cdot \underline{b} = 0.$$

$$(3\underline{c} - \underline{b}) \cdot (3\underline{c} + \underline{b}) = 0 \quad (5)$$

(25)



$$\tan \theta = \frac{\sqrt{3}a}{3a} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \pi/6 \quad (5)$$

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$$\vec{E} = -30\sqrt{3} \times \sqrt{3}a - 5 \times 2\sqrt{3}a \times \sin \frac{\pi}{3} + 10 \times 2\sqrt{3}a \sin \frac{\pi}{3} \quad (10)$$

$$+ 15 \times 4\sqrt{3}a \sin \frac{\pi}{3}$$

$$= -90a - 15a + 30a + 90a$$

$$= 15a \neq 0. \quad (5)$$

$\therefore$ system cannot be in eq^m. (5)

(ii) If the sy^m reduces to a couple;

$$X = 0 \quad (5) \text{ and } Y = 0. \quad (5)$$

$$\rightarrow X = -30\sqrt{3} - 5 \cos \frac{\pi}{6} + 10 \cos \frac{\pi}{6} + 15 \cos \frac{\pi}{6} \quad (10)$$

$$-30\sqrt{3} - 5 \times \frac{\sqrt{3}}{2} + 10 \times \frac{\sqrt{3}}{2} + 15 \times \frac{\sqrt{3}}{2} - \lambda \cos \frac{\pi}{6} = 0$$

$$-30\sqrt{3} - 5 \times \frac{\sqrt{3}}{2} + 10 \times \frac{\sqrt{3}}{2} + 15 \times \frac{\sqrt{3}}{2} - \lambda \times \frac{\sqrt{3}}{2} = 0$$

$$\uparrow Y = -M + 5 \sin \frac{\pi}{6} + 10 \sin \frac{\pi}{6} - 15 \sin \frac{\pi}{6} - \lambda \sin \frac{\pi}{6} = 0. \quad (5)$$

$$-M - \lambda \sin \frac{\pi}{6} = 0 \quad (10)$$

$$M = 40 \times \frac{1}{2} = 20.$$

$$\underline{M = 20} \quad (5)$$

(iii) Resolved component along $\vec{AD} = 10 \text{ N} \quad (5)$

Resolved component perpendicular to $AD = 0 \quad (5)$

$$\therefore \downarrow M + \lambda \sin \frac{\pi}{6} = 10 \quad (5) \quad \vec{X} = 0 \quad (5)$$

$$M = 10 - \lambda \sin \frac{\pi}{6}$$

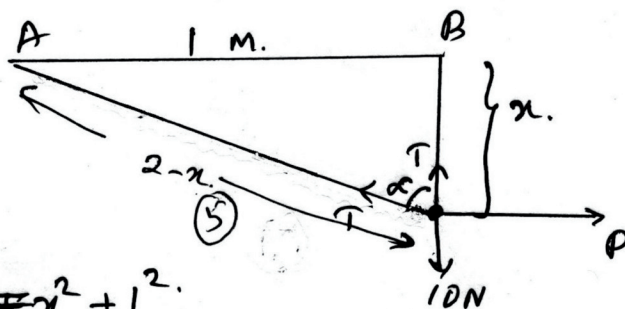
$$= 10 + 40 \times \frac{1}{2}$$

$$\underline{\underline{\lambda = -40}} \quad (5)$$

$$M = 30 \text{ N} \quad (5)$$

100

(15) (b).



$$(2-x)^2 = x^2 + 1^2$$

$$4x = 3$$

$$x = \frac{3}{4} \text{ m. (5)}$$

$$\therefore 2-x = 2 - \frac{3}{4} = \frac{5}{4} \text{ m.}$$

$$\therefore \cos \alpha = \frac{x}{2-x} = \frac{3}{5} \text{ (5)}$$

$$\sin \alpha = \frac{1}{2-x} = \frac{4}{5} \text{ (5)}$$

$$\uparrow ; T + T \cos \alpha - 10 = 0 \text{ (10)}$$

$$T + T \times \frac{3}{5} = 10$$

$$T = \frac{50}{8} \text{ N (5)}$$

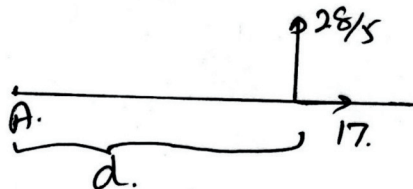
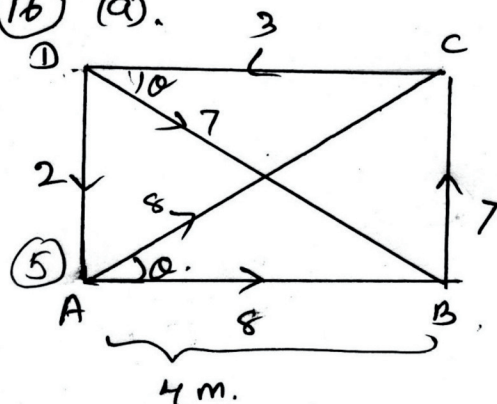
$$\rightarrow P = T \sin \alpha \text{ (10)}$$

$$= \frac{50}{8} \times \frac{4}{5} = 5 \text{ N.}$$

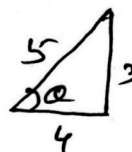
$$\underline{\underline{P = 5 \text{ N. (5)}}$$

50

(16) (a).



$$\tan \alpha = \frac{3}{4} \text{ (5)}$$



$$\begin{aligned} X &= 8 - 3 + 8 \cos \alpha + 7 \cos \alpha \\ &= 8 - 3 + 15 \cos \alpha \text{ (10)} \\ &= 8 - 3 + 15 \times \frac{4}{5} \end{aligned}$$

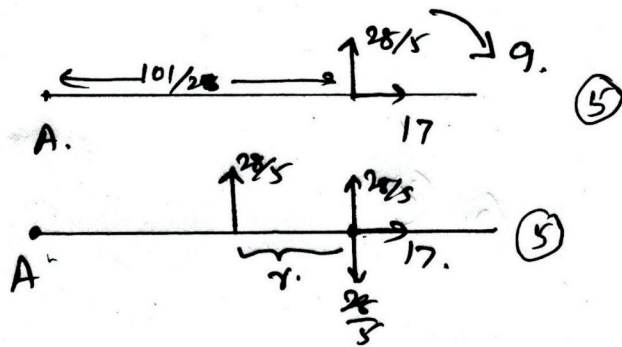
$$\underline{\underline{X = 17 \text{ (5)}}$$

$$\uparrow Y = -2 + 7 + 8 \sin \alpha - 7 \sin \alpha \text{ (10)}$$

$$\underline{\underline{Y = \frac{28}{5} \text{ (5)}}$$

$$\begin{aligned} \text{A) } \frac{28}{5} d &= 7 \times 4 + 3 \times 3 - 7 \cos \alpha \times 3 \text{ (10)} \\ &= 28 + 9 - 7 \times 3 \times \frac{4}{5} \end{aligned}$$

$$\frac{28d}{5} = \frac{101}{5} \Rightarrow d = \frac{101}{28} \text{ cm. (5)}$$



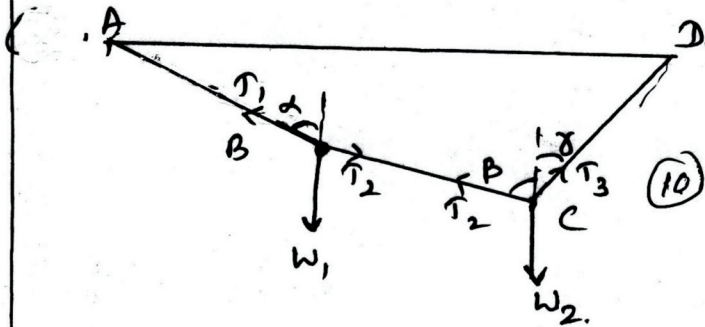
$$\frac{28}{5} \times r = 9 \quad (5)$$

$$r = \frac{45}{28} \text{ cm.} \quad (5)$$

$$\therefore \text{distance from A} = \frac{101}{28} - \frac{45}{28} = \underline{\underline{2 \text{ m}}} \quad (5)$$

80

(b) For the Lami's theorem. 10



Applying Lami's theorem

at B.

$$\frac{W_1}{\sin(180 - \beta + \alpha)} = \frac{T_2}{\sin(180 - \alpha)} \quad (10)$$

$$(5) \frac{W_1}{\sin(\beta - \alpha)} = \frac{T_2}{\sin \alpha} \quad (5)$$

at C.

$$\frac{W_2}{\sin(\beta + \gamma)} = \frac{T_3}{\sin(180 - \gamma)} \quad (10)$$

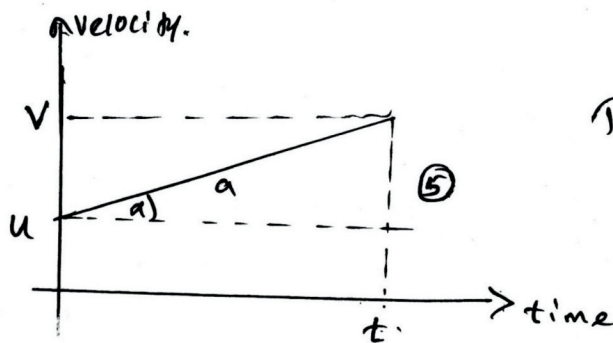
$$(5) \frac{W_2}{\sin(\beta + \gamma)} = \frac{T_3}{\sin \gamma} \quad (5)$$

From (1) and (2):

$$\frac{W_1 \sin \alpha}{\sin(\beta - \alpha)} = \frac{W_2 \sin \gamma}{\sin(\beta + \gamma)} \quad (5)$$

$$\frac{W_1}{W_2} = \frac{\sin \gamma \sin(\beta - \alpha)}{\sin \alpha \sin(\beta + \gamma)} \quad (5)$$

60



$$\text{slope} = a = \frac{v-u}{t} \quad (5)$$

$$S = \left(\frac{u+v}{2}\right)t \quad (5)$$

For the proof of: $V = u + at$
 $S = ut + \frac{1}{2}at^2$
 $V^2 = u^2 + 2as$ } (15) marks.

30

(b).

Let T is the time for meet two particles.



for 2nd particle: $S = ut + \frac{1}{2}at^2$ ↑

$$H = u(T-t) - \frac{1}{2}g(T-t)^2 \quad (10)$$

for 1st particle,

$$H = uT - \frac{1}{2}gT^2 \quad (10)$$

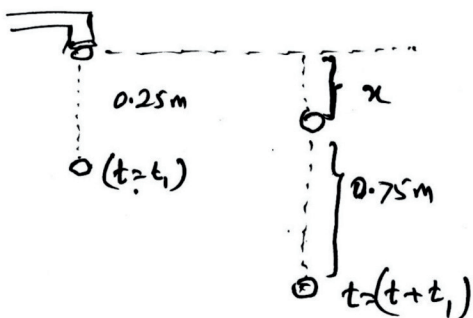
$$\therefore uT - \frac{1}{2}gT^2 = u(T-t) - \frac{1}{2}g(T-t)^2 \quad (10)$$

$$uT - \frac{1}{2}gT^2 = uT - ut - \frac{1}{2}g(T^2 + t^2 - 2Tt) \quad (10)$$

$$T = \left(\frac{u}{g} + \frac{t}{2}\right) \quad (5)$$

$$\text{from } (1) \Rightarrow H = u\left[\frac{u}{g} + \frac{t}{2}\right] - \frac{1}{2}g\left(\frac{u}{g} + \frac{t}{2}\right)^2 = \frac{4u^2 - g^2t^2}{8g} \quad (60)$$

(c).



for the 1st drop: ↓

$$(x+0.75) = \frac{1}{2} \times 10(t_1+t_2)^2 \quad (10) \quad (2)$$

for the second drop: ↓

$$x = \frac{1}{2} \times 10t_2^2 \quad (3) \quad (10)$$

from (2); and (3).

$$0.75 = 5(t_1+t_2)^2 - 5t_2^2 \quad (10)$$

$$t_2 = \frac{\sqrt{5}}{10} \text{ sec} \quad (5)$$

$$\text{from } (3) \Rightarrow x = \frac{1}{2} \times 10 \times \frac{5}{10} = \frac{1}{4} \text{ m.}$$

for the 1st water drop:

$$S = ut + \frac{1}{2}at^2 \quad \downarrow$$

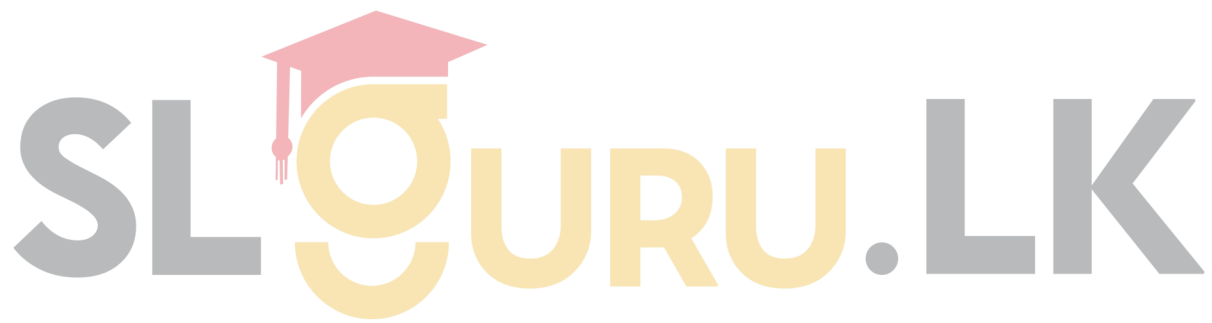
$$(10) \quad 0.25 = \frac{1}{2} \times 10 t_1^2 \quad (1)$$

$$\therefore t_1 = \frac{1}{2\sqrt{5}} \quad (5)$$

$\therefore$ total distance traveled by 1st drop $y = \frac{1}{4} + \frac{3}{4} = 1 \text{ m.}$

60

“Education is the foundation upon which we build our future.” —
Christine Gregoire



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